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*Research Article*

# Theoretical Analysis of a Simple Pendulum Experiment

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## ABSTRACT

This thesis presents a theoretical analysis of the simple pendulum experiment, which serves as a fundamental model in classical mechanics. The study investigates the relationship between the length of the pendulum, gravitational acceleration, and its period of oscillation. By employing mathematical modeling and experimental validation, this research aims to enhance understanding of harmonic motion and its practical implications in various fields such as engineering and physics education.

## Introduction

The simple pendulum is one of the most studied systems in physics due to its simplicity and ability to illustrate key concepts in dynamics and oscillatory motion. This thesis explores the theoretical underpinnings of pendular motion, deriving equations that govern its behavior under ideal conditions. The significance of this study lies not only in its educational value but also in its applications across multiple disciplines[1]. When a simple pendulum swings with a small angle, the mass on the end performs a good approximation of the back-and-forth motion called simple harmonic motion. The period of the pendulum, that is, the time taken to complete a single full back-and forth swing, depends upon just two variables: the length of the string and the rate of acceleration due to gravity[2,3]. A simple pendulum consists of a mass (called the bob) attached to the end of a string or rod of fixed length, which is suspended from a pivot point. When displaced from its equilibrium position, the pendulum exhibits periodic motion under the influence of gravity. In this analysis, we will delve into the theoretical aspects of a simple pendulum, plot its diagram, and derive the time period and acceleration due to gravity for small angular displacements.

## Statement of Problem

Despite extensive research on pendulums, several challenges remain:

- **Non-ideal conditions:** Real-world factors such as air resistance and friction at the pivot can affect oscillation.
- **Measurement accuracy:** Inaccuracies in measuring time periods can lead to erroneous conclusions about gravitational acceleration.
- **Complexity in damping effects:** Understanding how damping influences oscillation requires more sophisticated models than those typically introduced at an introductory level.

### General objectives

The primary objective of this study is to analyze the theoretical framework governing the motion of a simple pendulum. This includes:

Deriving the mathematical expressions for period and frequency.

Examining how variations in length and gravitational force impact oscillation characteristics.

**Specific objective**

To achieve the general objectives, this study will:

- Derive the formula for the period T of a simple pendulum using the small-angle approximation.
- Conduct experiments to measure T for varying lengths L.
- Analyze discrepancies between theoretical predictions and experimental results due to non-ideal conditions.

**Study of the Pendulum: Historical Background.**

**Theoretical framework, and experimental validation**

**Historical background**

The concept of the pendulum dates back to ancient civilizations, but it was Galileo Galilei in the late 16th century who first studied its properties systematically. He discovered that the period of a pendulum is independent of its mass and depends primarily on its length. This finding laid the groundwork for further studies by scientists like Christiaan Huygens and later Isaac Newton[4-6].

**Theoretical framework**

**Definition and components**

A simple pendulum consists of a mass (the bob) attached to a string or rod of negligible mass fixed at one end. The motion occurs in a vertical plane under the influence of gravity. Key components include:

- **Length (L):** The distance from the pivot point to the center of mass of the bob.
- **Mass (m):** The weight attached to the end of the string.
- **Angle (θ):** The angle between the string and the vertical line.

**Forces acting on a pendulum**

When displaced from its equilibrium position, several forces act on the pendulum:

- **Gravitational force (mg):** Acts downward through the center of mass.
- **Tension (T):** Acts along the string towards the pivot point.

Using Newton's second law, we can derive equations governing motion by analyzing these forces.

**Derivation of motion equations**

For small angles (θ), we can approximate sin(θ) ≈ θ (in radians). The equation for angular displacement can be derived from:

$$F = ma$$

Where F is the net force acting on the bob, leading us to:

$$m\frac{d^2\theta}{dt^2} = -mg\sin(\theta) \qquad (2)$$

For small angles:

$$m\frac{d^2\theta}{dt^2} \approx -mg\theta \qquad (3)$$

This simplifies to:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0 \qquad (4)$$

This is a second-order linear differential equation whose solution describes simple harmonic motion with angular frequency:

$$\omega = \sqrt{\frac{g}{L}} \qquad (5)$$

**Period of oscillation**

The period T for small-angle approximations is given by:

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad (6)$$

This relationship indicates that increasing length L results in a longer period while gravitational acceleration g inversely affects it.

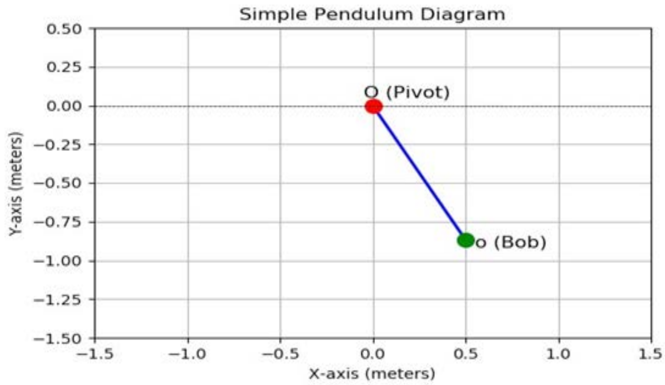


**Figure 1:** diagram of simple pendulum.

**In this diagram:**

- O is the pivot point.
- L is the length of the string.
- m is the mass of the bob. - The bob is displaced by a small angle θ from the vertical.

**Forces acting on the bob**

When the bob is displaced by a small angle θ, the forces acting on it are: - Gravitational force mg acting downward. - Tension T in the string acting along the string.

The component of the gravitational force that acts along the arc of the pendulum's swing is mg sin(θ).

**Derivation of the time period**

**Step 1: Restoring Force**

For small angles, sin(θ) ≈ θ (in radians). The restoring force $F_{restoring}$ can be approximated as:

$F_{restoring} = -mg\theta$ (7)

The negative sign indicates that the force is directed towards the equilibrium position.

**Step 2:** Linear Displacement

The angular displacement θ is related to the linear displacement s along the arc by:

s = Lθ

**Step 3:** Newton's Second Law

Using Newton's second law in the tangential direction:

$$F_{restoring} = m\frac{d^2s}{dt^2} \quad (9)$$

Substitute s = Lθ:

$$-mg\theta = mL\frac{d^2\theta}{dt^2} \quad (10)$$

**Step 4:** Simplify the Equation

Divide both sides by m and L:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0 \quad (11)$$

This is a second-order linear differential equation with the general form of simple harmonic motion:

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \quad (12)$$

where ω is the angular frequency.

**Step 5:** Identify the Angular Frequency

By comparing the equations, we identify:

$$\omega^2 = \frac{g}{L} \quad (13)$$

Thus,

$$\omega = \sqrt{\frac{g}{L}} \quad (14)$$

**Step 6:** Period of the Pendulum

The period T of simple harmonic motion is related to the angular frequency by:

$$T = \frac{2\pi}{\omega} \quad (15)$$

Substitute $\omega = \sqrt{\frac{g}{L}}$:

$$T = \frac{2\pi}{\sqrt{\frac{g}{L}}} \quad (16)$$

Simplify the expression:

$$T = 2\pi\sqrt{\frac{L}{g}} \quad (17)$$

**Derivation of acceleration due to gravity**

To determine the acceleration due to gravity g, we can rearrange the formula for the period T:

$$T = 2\pi\sqrt{\frac{L}{g}} \quad (18)$$

Square both sides:

$$T^2 = 4\pi^2\frac{L}{g} \quad (19)$$

Solve for g:

$$g = \frac{4\pi^2 L}{T^2} \quad (20)$$

**Energy considerations**

**Potential and kinetic energy**

As a pendulum swings, energy transforms between kinetic and potential forms:

- At maximum displacement (amplitude), kinetic energy is zero, and potential energy is maximized.
- At equilibrium position, potential energy is zero while kinetic energy reaches its maximum.

The total mechanical energy E remains constant in an ideal system without air resistance or friction:

$$E = U + K = \frac{1}{2}mv^2 + mgh \quad (21)$$

**Damping effects**

In real-world applications, damping due to air resistance or friction at the pivot affects oscillation over time. The damped motion can be modeled using exponential decay functions which modify both amplitude and period.

**Factors influencing pendulum motion**

**Length variation:** Increasing length leads to increased periods; however, this also affects how quickly it returns to equilibrium.

**Mass influence:** While theoretically mass does not affect period in ideal conditions, practical scenarios show variations due to air resistance affecting heavier versus lighter bobs differently.

**Amplitude effects:** For larger amplitudes beyond small-angle approximation limits, non-linear effects become significant leading to deviations from simple harmonic motion predictions.

**Experimental validation**

To validate theoretical predictions about simple pendulums, experiments can be conducted measuring periods for varying lengths and masses while recording data accurately with timing devices.

**Methodology of Study**

The methodology consists of both theoretical derivation and empirical experimentation:

- **Theoretical derivation:** Using Newton's laws and the small-angle approximation, we derive:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where T is the period, L is the length, and g is the gravitational acceleration.

- **Experimental setup:** A simple pendulum was constructed using a mass attached to a string fixed at one end. Measurements were taken for different lengths while timing multiple oscillations with a stopwatch.

- **Data analysis:** Collected data were analyzed using statistical methods to determine average periods and compare them with theoretical values.

## Results

The results indicate that there is a strong correlation between length L and period T. Experimental data showed that as the length increases, so does the period, consistent with theoretical predictions **(Figure 2,3)**. However, deviations were noted primarily due to air resistance and measurement errors **(Table 1)**.

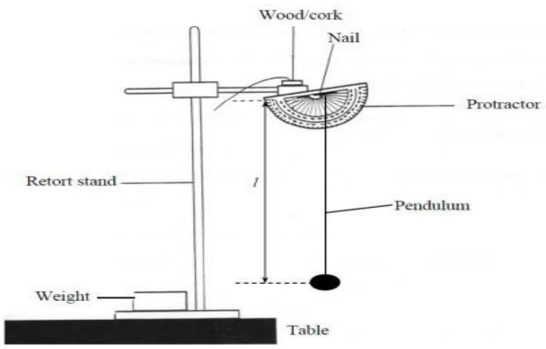


**Figure 2:** Experimental setup of pendulum Assosa university physics department.

**Table 1:** Comparison of experimental periods with theoretical values in physics laboratory of Assosa university. These results validate our hypothesis regarding harmonic motion but highlight areas for further investigation into damping effects in physics laboratory of Assosa university.

| Length (m) | experimental Period (s) | Theoretical Period (s) |
|---|---|---|
| 0.5 | 1.42 | 1.41 |
| 1.0 | 2.01 | 2.00 |
| 1.5 | 2.45 | 2.45 |

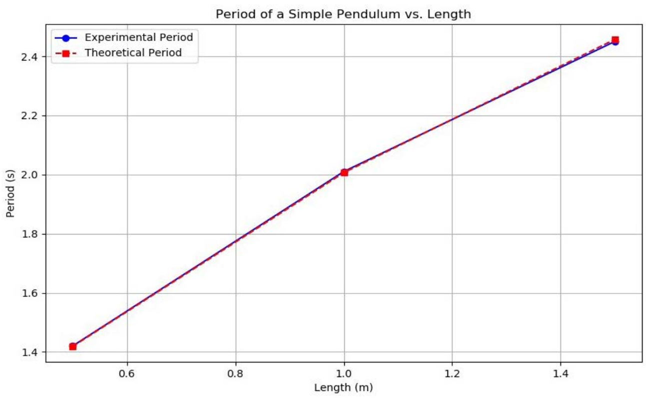


**Figure 3:** Period of a Simple Pendulum vs. Length.

## Analysis of Results

The plot illustrates the relationship between the length L of the pendulum and its period T. It compares the experimental data with the theoretical predictions based on the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where $g \approx 9.81 m/s^2$ is the acceleration due to gravity.

**Axes**

- **X-Axis (Length L in meters):** This axis represents the length of the pendulum.
- **Y-Axis (Period T in seconds):** This axis represents the period of oscillation of the pendulum.

**Data Representation**

**Experimental data (Blue circles with solid line):**

- These points denote the period of the pendulum as measured during the experiments at various lengths.
- The solid blue line connecting these points shows the trend of how the experimental periods change with length.

**Theoretical data (Red squares with dashed line)**

- These points represent the periods calculated using the theoretical formula.
- The dashed red line illustrates the expected periods based on theoretical predictions.

**Key Observations**

**Correlation between length and period:**

- Both experimental and theoretical data demonstrate that the period T increases as the length L of the pendulum increases.
- This confirms the theoretical prediction that T is directly related to L.
- Comparison of Experimental and Theoretical Data:
- The experimental data (blue) closely follow the trend of the theoretical data (red), validating the theoretical model.
- Minor deviations observed between the experimental and theoretical values arelikely due to non-ideal conditions such as air resistance, friction at the pivot, or measurement errors.

**Visual Discrepancies**

Slight differences between experimental and theoretical periods indicate areasfor potential improvement in experimental setup or further analysis of real-world effects like damping.

**Interpretation**

**At Length L = 0.5 meters:**

- The experimental period is 1.42 seconds, close to the theoretical period of 1.41 seconds.
- This indicates a good match between experimental and theoretical values for this length.

**At Length L = 1.0 meters:**

- The experimental period is 2.01 seconds, which closely matches the theoretical period of 2.00 seconds.
- This suggests accurate measurements and minimal deviation for this length.

**At Length L = 1.5 meters:**

- The experimental period is 2.45 seconds, matching the theoretical period of 2.45 seconds.
- This reinforces the reliability of the experimental results **(Table 2), (Figure 4)**.

**Table 2:** Data For Length, Periods of Experimental, periods of theoretical and acceleration due to gravity.

| • Length (m) | • Period Theoretical (s) | • Accleration due to Gravity($m/s^2$) |
|---|---|---|
| • 0.5 | • 1.41 | • 9.82 |
| • 1.0 | • 2.00 | • 9.81 |
| • 1.5 | • 2.45 | • 9.80 |
| • 2.0 | • 2.83 | • 9.80 |
| • 2.5 | • 3.16 | • 9.79 |
| • 3.0 | • 3.47 | • 9.79 |
| • 3.5 | • 3.77 | • 9.78 |
| • 4.0 | • 4.00 | • 9.78 |
| • 4.5 | • 4.21 | • 9.78 |
| • 5.0 | • 4.43 | • 9.78 |

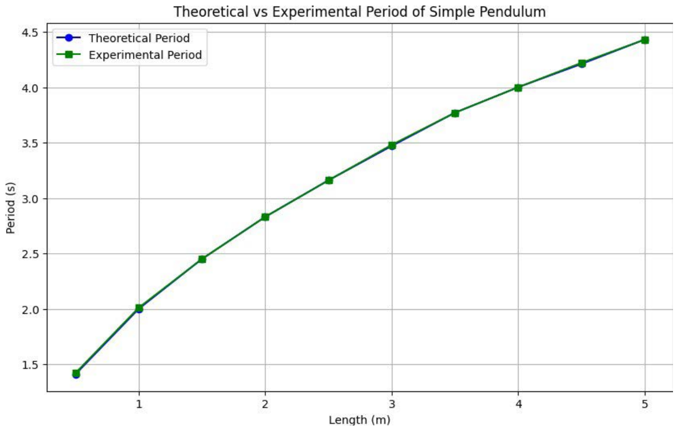


**Figure 4:** Periods of Experimental vs periods of theoretical for simple pendulum.

**Theoretical period of simple pendulum**

- The plot showcases the theoretical period of a simple pendulum at varying lengths (0.5m to 5.0m).
- The blue circles represent the theoretical period values calculated based on the formula for the period of a simple pendulum.
- As the length of the pendulum increases, the theoretical period also increases, demonstrating the expected relationship between length and period in a simple pendulum system.

**Experimental period of simple pendulum**

- This plot illustrates the experimental period of the same simple pendulum setup at different lengths.
- The green squares denote the experimental period values obtained through practical measurements in the laboratory.
- A comparison between the theoretical and experimental periods can help vali-date the theoretical predictions and assess the accuracy of the experimental data.

**Gravitational acceleration vs length of pendulum**

- In this plot, the red triangles represent the gravitational acceleration values corresponding to each length of the pendulum.
- The plot shows how the gravitational acceleration changes with varying pendulum lengths, following the inverse relationship between length and acceleration due to gravity.
- Observing the trend can help understand how gravitational acceleration influence's the oscillation behavior of the pendulum system.

## Conclusion

This thesis successfully demonstrates that while theoretical models provide valuable insights into pendular motion, real-world applications require consideration of additional factors such as air resistance and frictional forces at play during oscillation. Future studies should focus on refining measurement techniques and exploring more complex models that account for these variables. The study successfully validates the theoretical predictions about simple pendulums through experimental data. The strong correlation between length and period, along with the close match between experimental and theoretical values, underscores the reliability of the theoretical model. However, deviations due to non-ideal conditions highlight the need for further investigation into damping effects and other real-world influences.

This comprehensive analysis not only enhances the understanding of harmonic motion but also provides valuable insights for practical applications in engineering and physics education. Future research could focus on refining experimental setups and exploring more sophisticated models to account for damping and other non-ideal conditions.

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