

Method of Constructing Reference Systems of Motion Relative to Scalar Motion

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ABSTRACT

Up to now, the Galilean and Lorentz transformations have been widely used in practice to study motion and dynamic processes. The choice of one or another transformation depends on the values of the velocities of the motions under study. The Galilean transformation is used to study low-velocity motions, while the Lorentz transformation, as is known, was adopted as a basis for constructing the special theory of relativity and is used to study high-velocity motions. In this case, the Lorentz transformation sets an upper velocity limit equal to the speed of light. The article notes that binding the upper velocity limit of the processes under study to the speed of an individual physical phenomenon may not always be fully justified.

This paper proposes a method for constructing motion reference systems based on an analysis of energy conservation laws. The constructed motion reference systems satisfy the requirement for invariance of intervals between events, but are not strictly tied to the speed of light. It is shown that scalar motion, the speed of which is taken as a basis for studying any other motions, does not necessarily have to have a strictly fixed upper limit value.

Keywords: Motion; Space; Frame of reference; Speed; Energy; Theory of relativity

Introduction

In various fields of physics and engineering sciences, inertial reference systems (IRS) of motion based on the Galilean and Lorentz transformations are currently widely used in practice to solve theoretical and applied problems related to motion¹⁻⁹. The Galilean transformation is used to study low-speed motion, while the Lorentz transformation, as is known, was adopted as a basis for constructing the special theory of relativity and is used to study high-speed motion. The main disadvantage of the Galilean transformation is that it does not fulfill the basic requirement of the theory of relativity on the invariance of intervals between

events. However, at low speeds relative to the speed of light, the Lorentz transformation practically coincides with the Galilean transformation. At the same time, to this day, in a number of publications¹⁰⁻¹³, the authors express doubts about the correctness of some conclusions of the special theory of relativity related to the interpretation of the Lorentz transformation. In¹⁴⁻²³, a method was proposed for studying the motion of arbitrary charged particles in electric and magnetic fields relative to the motion of a scalar (central) charged particle. In general, the analysis of the possibilities for improving the methods for studying motion and dynamic processes is a problem that does not lose its relevance.

This paper examines methods for constructing reference systems for motions that satisfy the requirement for invariance of intervals between events, but are not strictly tied to the speed of light.

2. Materials and Methods

As is known, in 1904 G. Lorentz was the first to use the transformation named after him to solve electrodynamic problems [23] and in 1905 A. Einstein published an article²⁴, which was accepted as the basis for the theory of relativity. Later, several methods were proposed for obtaining the Lorentz transformation based on the Galilean transformations given below, which relate the coordinates x , y , z and time t of the coordinate system (reference) K with the coordinates x' , y' , z' and time t' of the reference system K' moving along the axis x of the coordinate system K with the speed V ,

$$x = x' + Vt', \quad y = y', \quad z = z', \quad t = t' \quad (1)$$

Galileo's frame of reference (1) is inertial and the velocity V is assumed to be constant.

In the theory of relativity, the postulate is introduced that the speed of light c is the maximum speed of motion in space and the Lorentz transformation has the following form

$$x = \frac{x' + Vt'}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad y = y', \quad z = z', \quad t = \frac{t' + \frac{V}{c^2}x'}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (2)$$

The method of constructing reference systems of motions, which is proposed in this article, is based on the analysis of the law of conservation of energies of a moving material body and the environment.

The motion of any material body in space is carried out under the influence of force action. At zero value (absence) of force action, the speed of the body in space is equal to zero. Therefore, any inertial motion with a non-zero value of speed is a consequence of the obtained (initial) force action in the absence of other acting forces on the moving object.

Let us now consider the process in which the force energy of space W_F exerts a force action on a material body, resulting in the movement of the body with kinetic energy W_{kin} . The force energy of space or energy that has the potential for a force action on a material body in space can be represented as a function of

the coordinates of space $W_F = W_F(\vec{R})$, where $\vec{R}$ is the radius vector that determines the coordinates of an arbitrary point in space. The energy of motion of a material body (kinetic energy) directly depends on the mass of the material body m and the speed of its motion $\vec{V}$. Therefore, kinetic energy can be represented as a function of the product of the mass and the speed of motion of the material body $W_{kin} = W_{kin}(m, \vec{V}) = W_{kin}(m\vec{V})$.

For motion in a closed space, the law of conservation of energy will be fulfilled under the condition

$$\frac{dW_{kin}}{dt} = \frac{dW_F}{dt} \quad (3)$$

The derivatives included in (3) can be represented as

$$\frac{dW_{kin}}{dt} = \frac{dW_{kin}}{d(m\vec{V})} \frac{d(m\vec{V})}{dt}, \quad \frac{dW_F}{dt} = \frac{dW_F}{d\vec{R}} \frac{d\vec{R}}{dt} \quad (4)$$

Taking into account (4), equation (3) has the form

$$\frac{dW_{kin}}{d(m\vec{V})} \frac{d(m\vec{V})}{dt} = \frac{dW_F}{d\vec{R}} \frac{d\vec{R}}{dt} \quad (5)$$

From equation (5) it follows

$$\frac{\frac{dW_{kin}}{d(m\vec{V})} \frac{d(m\vec{V})}{dt}}{\vec{V} \frac{dW_F}{d\vec{R}}} = \frac{\frac{dW_{kin}}{d(m\vec{V})}}{\vec{V}} \cdot \frac{d}{dt} \left(m \frac{d\vec{R}}{dt} \right) = 1 \quad (6)$$

Considering that kinetic and potential energies depend on different measurement systems and displaying the acceleration of motion in a spatial coordinate system, we obtain

$$\frac{dW_{kin}}{d(m\vec{V})} = \vec{V} \quad (7)$$

$$\frac{d}{dt} \left(m \frac{d\vec{R}}{dt} \right) = m \frac{d^2 \vec{R}}{dt^2} = \frac{dW_F}{d\vec{R}} = \nabla W_F = \vec{F} \quad (8)$$

From equation (7) we have

$$dW_{kin} = \vec{V} d(m\vec{V}) = \frac{1}{2} d(mV^2) \quad (9)$$

from which follows the well-known classical equation of kinetic energy

$$W_{kin} = \frac{m}{2} \int_0^V d(V^2) = \frac{mV^2}{2} \quad (10)$$

Here it is assumed that the mass of a moving particle (or material body) is a constant.

Equation (8) is Newton's second law. Note that the kinetic energy equation is usually derived based on Newton's second law. Here equations (8) and (10) are obtained separately and simultaneously, based on the law of conservation of energy.

3. Results

To analyze the possibilities of constructing reference systems for motions, we consider equation (10) as a sum of integrals

$$W_{kin} = \frac{m}{2} \int_0^V d(V^2) = \frac{m}{2} \left[\int_0^V d(V^2) + \int_V^V d(V^2) \right] = \frac{m}{2} [V^2 + (V_s^2 - V^2)] \quad (11)$$

Having designated

$$V'^2 = V_s^2 - V^2 \quad (12)$$

we move from the initial frame of reference K with the coordinate system x, y, z and time t to the moving frame of reference K' with coordinates x', y', z' and time t' . Here the relationship between coordinates and velocities in the considered frames of reference will satisfy the equations

$$\frac{d\vec{R}_s}{dt_s} = \vec{V}_s, \quad \frac{d\vec{R}}{dt_s} = \vec{V}, \quad \frac{d\vec{R}'}{dt_s} = \vec{V}' \quad (13)$$

In (12) and (13) and denote, respectively, the time and speed of movement, which is selected as a scalar movement, the speed of movement in the original coordinate system.

It is easy to see that equation (12) can be represented as

$$V' = V_s \sqrt{1 - \frac{V^2}{V_s^2}} \quad (14)$$

Note that when choosing a value of scalar velocity equal to the speed of light $V_s = c$, equation (14) takes the form

$$V' = c \sqrt{1 - \frac{V^2}{c^2}}$$

From equations (13) and (14) it follows

$$\frac{dR'}{dt} = \frac{dR_s}{dt_s} \sqrt{1 - \frac{V^2}{V_s^2}} \quad (15)$$

Here V and V_s are constant values. To derive the inertial reference frame, equation (15) can be represented as

$$\frac{R'}{t_s} = \frac{R_s}{t_s} \sqrt{1 - \frac{V^2}{V_s^2}} \quad (16)$$

From equations (15) and (16) it follows

$$dR' = dR_s \sqrt{1 - \frac{V^2}{V_s^2}}, \quad R' = R_s \sqrt{1 - \frac{V^2}{V_s^2}} \quad (17)$$

If we represent equation (16) in the form

$$\frac{R'}{t_s \sqrt{1 - \frac{V^2}{V_s^2}}} = \frac{R_s}{t_s}$$

then under the condition $R' = R_s$ we obtain

$$t' = t_s \sqrt{1 - \frac{V^2}{V_s^2}} \quad (18)$$

Equations (18) and (14) reflect the regularity that the times of passage t' and t_s of equal distances at different speeds of movement V' and V_s must differ.

From equations (17) and (18) it follows

$$\frac{R'}{t'} = \frac{R_s \sqrt{1 - \frac{V^2}{V_s^2}}}{t_s \sqrt{1 - \frac{V^2}{V_s^2}}} = \frac{R_s}{t_s} = V_s$$

Note that in a more expanded version, this expression can be presented as a condition for choosing a scalar velocity as an invariant that links the values of spatial coordinates and times in all reference systems of motion

$$\vec{V}_s = \frac{\vec{R}_s}{t_s} = \frac{\vec{R}}{t} = \frac{\vec{R}'}{t'} \quad (19)$$

Taking into account (15) and (17), equation (19) can also be represented as

$$\vec{V}_s = \frac{d\vec{R}_s}{dt_s} = \frac{d\vec{R}}{dt} = \frac{d\vec{R}'}{dt'}$$

From equation (19) it follows that in all the reference systems under consideration the value of the kinetic energy of the motion under study remains an invariant quantity. This ensures the invariance of the intervals between events.

Now let us consider the transformation of coordinates, adopting in the new reference system

$$x = x_s + V t_s, \quad y = y', \quad z = z' \quad (20)$$

Taking into account the condition that scalar motion in the considered frame of reference exists only in the x direction, for which $x' = R'(x', 0, 0)$ and $x_s = R_s(x_s, 0, 0)$, we use equation (17) in the form

$$x' = x_s \sqrt{1 - \frac{V^2}{V_s^2}}, \quad t' = t_s \sqrt{1 - \frac{V^2}{V_s^2}} \quad (21)$$

Then, from (18), (20) and (21) we obtain

$$x = \frac{x' + V t'}{\sqrt{1 - \frac{V^2}{V_s^2}}} \quad (22)$$

Taking into account (19) and the accepted direction of scalar motion x , time can be represented as

$$t = \frac{x}{V_s}, \quad t' = \frac{x'}{V_s}$$

then from equation (22) we have

$$t = \frac{t' + \frac{V}{V_s^2} x'}{\sqrt{1 - \frac{V^2}{V_s^2}}} \quad (23)$$

Thus, the transformation of IRS coordinates will have the following form

$$x = \frac{x' + Vt'}{\sqrt{1 - \frac{V^2}{V_s^2}}}, \quad y = y', \quad z = z', \quad t = \frac{t' + \frac{V}{V_s^2} x'}{\sqrt{1 - \frac{V^2}{V_s^2}}} \quad (24)$$

In the special case, for $V_s = c$, we obtain the Lorentz transformation (2).

Note that in transformation (24) the speed of scalar motion is not strictly tied to the speed of light. Depending on the characteristics of the processes being studied, the value of the scalar speed can be chosen either equal to the speed of light or to another limiting value of speed that most effectively describes the phenomena being studied.

To analyze the possibilities of constructing other systems of coordinate transformations that differ from the reference systems of motions (24), we consider equation (10) in the form of the following sum of integrals

$$W_{kin} = \frac{m}{2} \int_0^V d(V^2) = \frac{m}{2} \left[\int_0^{V_s} d(V^2) + \int_{V_s}^V d(V^2) \right] = \frac{m}{2} [V_s^2 + (V^2 - V_s^2)] \quad (25)$$

Here we denote (26)

$$V'^2 = V^2 - V_s^2 \quad (26)$$

and rewrite this expression as

$$V^2 = V_s^2 + V'^2 \quad (27)$$

Now, as above, we will move from the initial frame of reference K with the coordinate system x, y, z and time t to the moving frame of reference $'$ with coordinates x', y', z' and time t' . In equations (25) – (27), the relationship between coordinates and velocities in the frames of reference under consideration will satisfy the above equations (13).

From (13) and (27) it follows

$$\frac{dR}{dt} = \frac{dR_s}{dt_s} \sqrt{1 + \frac{V'^2}{V_s^2}} \quad (28)$$

Using equation (28) according to the above method we obtain

$$R = R_s \sqrt{1 + \frac{V'^2}{V_s^2}}, \quad t = t_s \sqrt{1 + \frac{V'^2}{V_s^2}} \quad (29)$$

Let us now consider the transformation of coordinates, adopting in the new reference system K'

$$x' = x_s - V't_s, \quad y' = y, \quad z' = z \quad (30)$$

Considering, as above, that the scalar motion is performed only along the axis x , from (29) it follows

$$x = x_s \sqrt{1 + \frac{V'^2}{V_s^2}} \quad (31)$$

Using equations (30), (31) and equalities $t = \frac{x}{V_s}$, $t' = \frac{x'}{V_s}$ we obtain

$$x' = \frac{x - V't}{\sqrt{1 + \frac{V'^2}{V_s^2}}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t - \frac{V'}{V_s^2} x}{\sqrt{1 + \frac{V'^2}{V_s^2}}} \quad (32)$$

An inertial reference frame based on transformation (32) can be successfully applied to study processes in which current velocities exceed given scalar velocity values.

4. Discussion

The new transformation (32) presented above was obtained based on equations (25) and (27). Transformations that differ in form from (32), but completely coincide in content, can be obtained by using equation (25) and equation (26) instead of equation (27). In this case, equation (26) can be further presented in two versions

$$V' = V_s \sqrt{\frac{V^2}{V_s^2} - 1} \quad (33)$$

Or

$$V' = V \sqrt{1 - \frac{V_s^2}{V^2}} \quad (34)$$

Thus, the number of different transformations in form can increase significantly.

Note that when studying dynamic processes in a given space, it is necessary to use Newton's second law (8) and take into account the law of conservation of energy in the form of (3) and (10). Therefore, when using expressions that connect different reference systems of motion, it is useful to note the connection of these expressions with the potential (force) energy of space. As an example, we give the following equations

$$\begin{aligned} \sqrt{1 - \frac{V^2}{V_s^2}} &= \sqrt{1 - \frac{W_F}{W_{Fs}}}, & \sqrt{1 + \frac{V'^2}{V_s^2}} &= \sqrt{1 + \frac{W_F'}{W_{Fs}}}, \\ \sqrt{\frac{V^2}{V_s^2} - 1} &= \sqrt{\frac{W_F}{W_{Fs}} - 1}, & \sqrt{1 - \frac{V_s^2}{V^2}} &= \sqrt{1 - \frac{W_{Fs}}{W_F}} \end{aligned}$$

Using the equation of motion of a charged particle with zero initial energy along a given trajectory, that is, setting a scalar (main) motion, allowed a group of Kazakhstani scientists led by Kelman V.M. and Yakushev E.M. to develop effective theories of electron mirrors and cathode lenses. In these theories, based on the principle of relativity of arbitrary motions relative to the main (scalar) motion, mathematical problems that led to

singularity of solutions were successfully solved. The author of this article, based on this method, developed and modified the theory of focusing charged particles in emission and emission-reflecting elements and units of electron-optical and ion-beam systems. Currently, these theories of electron mirrors and emission systems are widely used by scientists and specialists in many countries for applied research and design of new science-intensive devices and instruments.

Now let us consider the above coordinate transformation in the form (20). The first equation of the transformation (20) and equation (18) can be represented as

$$x = x_s + Vt_s = (V_s + V)t_s, \quad t' = \gamma t_s$$

Where,

$$\gamma = \sqrt{1 - \frac{V^2}{V_s^2}}$$

Taking these expressions into account, the coordinate transformation (20) takes the form

$$x = (V_s + V)t_s, \quad y = y', \quad z = z', \quad t_s = \frac{1}{\gamma} t'$$

Here it is clear that in the transformation under consideration there is a conversion factor γ only for time values t' and t_s . As noted above, the presence of this factor is due to the fact that the times of passing equal distances at different speeds of movement V' and V_s must differ.

Note that transformation (24) was derived based on the analysis of equations (12), (13) and (19). When deriving transformation (32), equation (27) was used instead of (12). These equations can also be used to construct dynamic reference systems of motions, with the help of which studies of motions in any coordinate system can be carried out taking into account the symmetries or asymmetries of the distribution of force fields.

The results of studies on the derivation of transformations based on equations (25) and (33), (25) and (34), on the derivation of coordinate transformations in a more general form, as well as a more detailed coverage of a number of current problems associated with the special theory of relativity will be presented by the author in the following works.

5. Conclusions

Up to now, only the Galilean and Lorentz transformations have practical application for studying motions and dynamic processes. In this case, the choice of one or another transformation depends only on the values of the velocities of the motions under study. The Lorentz transformation is still used only to study high-velocity motions. In this case, high velocities have an upper limit and are limited by the speed of light. Linking the upper limit of the velocity of the processes under study to the velocity of a separate physical phenomenon may not always be fully justified. In addition, in modern theoretical physics there are a number of problems associated with problems in the field of mathematics.

In this paper, a method for constructing reference systems for motions based on the analysis of energy conservation laws is proposed. It is shown that scalar motion, the velocity of which is taken as a basis for studying any other motions, does not necessarily have to have a strictly fixed upper limit value.

The article presents the derivation of two types of coordinate transformations (24) and (32), in which the upper limit of the velocity can be chosen depending on the characteristics of specific processes under study. In this case, the Lorentz transformation can be obtained as a special case of transformation (24) when choosing the value of the scalar velocity equal to the speed of light. In addition, it is noted that the author of the article has developed other transformations in private and generalized forms, which will be presented for publication later.

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